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Fixed Point Results for Generalized Weakly C- contractive Mappings in Ordered G-partial Metric Spaces

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Abstract

We introduced the class of generalized weakly C-contractive mappings in G-partial metric spaces by combining the characteristics of Hardy and Rogers maps with weak contraction maps. The existence and uniqueness of fixed point for those maps in ordered G-partial metric spaces are established. Examples are given to support the validity of our results. Our results generalize some results in the literature.

Keywords: Fixed points; generalized weakly C- contractive mappings; weakly C- contractive mappings; G-partial metric spaces; ordered G-partial metric spaces.

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1 Introduction

Metric fixed point theory has been a rigorous area of research in fixed point theory and applications. Several authors have worked on the generalization of the notion of metric space. In particular, Matthew [1] generalized the notion of metric space by introducing the concept of nonzero self distance. Mustafa and Sims [2] also extended the concept of metric to G-metric by assigning the real number to every triplet of an arbitrary set. Recently in [3], the concept of G-partial metric

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space is established by introducing the concept of nonzero self-distance to the notion of G-metric space. In the same reference, some fixed point results for contraction maps in ordered G-partial metric space are proved. In a decade, the existence of fixed points in ordered metric spaces was initiated by Ran and Reurings [4]. Olaleru et al. [5] established the uniqueness of fixed points for some Ciric-type contractive maps in ordered G-partial metric space. In this work we proved our results also in ordered G-partial metric space.

The following definitions and motivations are found in [3].

Definition 1.1 : Let X be a nonempty set, and let $G_p : X \times X \times X \to R^+$ be a function satisfying the following:

(G_p1) $G_p(x,y,z) \geq G_p(x,x,x) \geq 0$ for all $x,y,z, \in X$(small self distance),
(G_p2) $G_p(x,y,z) = G_p(x,x,y) = G_p(y,y,z) = G_p(z,z,x)$ iff $x = y = z$, (equality),
(G_p3) $G_p(x,y,z) = G_p(z,x,y) = G_p(y,z,x)$ (symmetry in all three variables),
(G_p4) $G_p(x,y,z) \leq G_p(x,a,a) + G_p(a,y,z) - G(a,a,a)$ (rectangle inequality).

Then the function G_p is called a G- partial metric and the pair (X, G_p) is called a G-partial metric space.

Example 1.2 : Let $X = R^+$ and a G-partial metric $G_p : X \times X \times X \to R^+$ can be defined with $G_p(x,y,z) = max\{x,y,z\}$ then (X, G_p) is a G-partial metric space.

We state the following definitions:

Definition 1.3: A sequence $\{x_n\}$ of points in a G-partial metric space (X, G_p) converges to some $a \in X$ if
$lim_{n\to\infty} G_p(x_n, x_n, a) = lim_{n\to\infty} G_p(x_n, x_n, x_n) = G_p(a,a,a)$.

Definition 1.4: A sequence $\{x_n\}$ of points in a G-partial metric spaces (X, G_p) is Cauchy if the numbers $G_p(x_n, x_m, x_l)$ converges to some $a \in X$ as n, m, l approach infinity.

The proof of the following proposition easily follows from definition.

Proposition 1.5: Let $\{x_n\}$ be a sequence in G-partial metric space X and $a \in X$. If $\{x_n\}$ converges to $a \in X$, then $\{x_n\}$ is a Cauchy sequence.

Definition 1.6: A G-partial metric space (X, G_p) is said to be complete if every Cauchy sequence in (X, G_p) converges to an element in (X, G_p). That is, $G_p(x,x,x) = lim_{n\to\infty} G_p(x_n, x, x) = lim_{n,m\to\infty} G_p(x_n, x_m, x_m)$.

Definition 1.7 [4]: Let $(X, \preceq)$ be a partially ordered set. Then two elements $x, y \in X$ are said to be totally ordered or ordered if they are comparable. i.e. $x \preceq y$ or $y \preceq x$.

Definition 1.8: Let X be a nonempty set. Then $(X, \preceq, G_p)$ is called an ordered G-partial metric space if the following conditions hold:

(i) G_p is a G-partial metric on X;
(ii) $\preceq$ is a partial order on X.

Lemma 1.9: Let (X, G_d) be a G-partial metric space, $T : X \to X$ be a given mapping. Suppose that T is continuous at $x_0 \in X$. Then, for each sequence $\{x_n\}$ in X, $x_n \to x_0 \Rightarrow Tx_n \to Tx_0$.

Definition 1.10 [6]: The function $\phi : [0, \infty) \to [0, \infty)$ is called an altering distance function, if the following properties are satisfied:

(1) ϕ is continuous and nondecreasing;
(2) $\phi(t) = 0$ if and only if $t = 0$.

Banach contraction mapping theorem is a well known result in fixed point theory. Though it has its drawback, that is the continunity of the map in the space. Kannan [7] introduced a class of map in which this condition (continunity of the map in the space) is not necessarily valid in proving the existence of fixed point for the map in metric space. Chatterjea [8] also introduced contractive maps different from that introduced in [7]. Choudhury [9] named the map introduced by Chatterjea after him as C-contraction map.

Definition 1.11 [8] (C-contraction): Let $T : X \to X$ where(X, d) is a metric space is called a C-contraction if there exists $0 < k < \frac{1}{2}$ such that for all $x, y \in X$ the following inequality holds:

$$d(Tx, Ty) \leq k[d(x, Ty) + d(y, Tx)]. \tag{1.1}$$

Other generalizations of Banach's contraction mapping included the weak contraction which was introduced in Hilbert space in [10]. The following is the corresponding definition in metric space given in [11].

Definition 1.12 [11] (weak - contraction) : A mapping $T : X \to X$ where (X, d) is a complete metric space is said to be weakly contractive if

$$d(Tx, Ty) \leq d(x, y) - \psi(d(x, y)). \tag{1.2}$$

where $x, y \in X$, $\psi : [0, \infty) \to [0, \infty)$ is continuous and nondecreasing, $\psi(x) = 0$ if and only if $x = 0$ and $lim_{x\to\infty}\psi(x) = \infty$.

Recently, Choudhury [9] introduced a generalized C-contraction which was termed weak C-contraction.

Definition 1.13 [9] : A mapping $T : X \to X$, where (X, d) is a complete metric space is said to be weakly C-contractive or weak C-contraction if for all $x, y \in X$,

$$d(Tx, Ty) \leq \frac{1}{2}[d(x, Ty) + d(y, Tx)] - \psi(d(x, Ty), d(y, Tx)). \tag{1.3}$$

where $\psi : [0, \infty)^2 \to [0, \infty)$ is a continuous mapping such that $\psi(x, y) = 0$ if and only if $x = y = 0$.

A more generalized C-contractive mapping is introduced by Hardy and Rogers [12].

Definition 1.14 [12] : Let (X, d) be a complete metric space and an operator $T : X \to X$ be a contractive mapping then there exist some numbers a, b, c, e and f, $a + b + c + e + f < 1$ such that for each $x, y \in X$,

$$d(Tx, Ty) \leq ad(x, y) + bd(x, Tx) + cd(y, Ty) + ed(x, Ty) + fd(y, Tx). \tag{1.4}$$

In [13], the existence of unique common fixed point for weakly compatible mappings in a metric space satisfying Hardy and Rogers contractive conditions is established. The existence of a unique fixed point for weak contraction mappings in G-metric spaces is proved in [14]. Eke [15] further established the existence of unique common fixed point for a pair of weakly compatable mappings satisfying weak contraction condition in G-metric space. Choudhury [9] established that weak C-contractive mapping actually have unique fixed point in complete metric spaces. The existence of a unique fixed point for weakly C-contractive mappings in ordered partial metric space is established in [6].

Theorem 1.1 [6] : Let $(X, \preceq)$ be a partially ordered set and suppose that there exists a partial metric on X such that (X,p) is complete. Let $T : X \to X$ be continuous nondecreasing mapping. Suppose that for comparable $x, y \in X$, we have

$$\psi(p(Tx,Ty)) \le \varphi(\frac{(p(x,Ty)+p(y,Tx)}{2}) - \phi(p(x,Ty),p(y,Tx)) \tag{1.5}$$

where ψ and φ are altering distance functions with

$$\psi(t) - \varphi(t) \ge 0. \tag{1.6}$$

for $t \ge 0$, and $\phi : [0,\infty)^2 \to [0,\infty)$ is a continuous function with $\phi(x,y) = 0$ if and only if $x = y = 0$. If there exists $x_0 \in X$ such that $x_0 \preceq Tx_0$, then T has a fixed point.

2 Main Results

In this work, we introduced a class of generalized weak C-contractive mapping in G-partial metric space by replacing the C-contraction map with Hardy and Rogers contractive map.

Definition 2.1: Let (X, G_p) be a G-partial metric space and $T : X \to X$ be a mapping. Then T is said to be generalized weakly C-contractive if for all $x, y \in X$, the following inequality holds:

$$\begin{aligned} G_p(Tx,Ty,Ty) \quad \le \quad & a_1G_p(x,y,y) + a_2G_p(x,Tx,Tx) + a_3G_p(y,Ty,Ty) \\ & +a_4G_p(x,Ty,Ty) + a_5G_p(y,Tx,Tx) - \phi(G_p(x,y,y), \\ & G_p(x,Tx,Tx), G_p(y,Ty,Ty), G_p(x,Ty,Ty), \quad G_p(y,Tx,Tx)) \end{aligned} \tag{2.1}$$

where a_1, a_2, a_3, a_4, $a_5 \in [0,1)$, $\sum_{i=1}^{5} a_i < 1$, and $\phi : [0,\infty)^5 \to [0,\infty)$ is a continuous function with $\phi(v,w,x,y,z) = 0$ if and only if $v = w = x = y = z = 0$.

Remarks 2.2: If $v = w = x = 0$, $a_1 = a_2 = a_3 = 0$, $a_4 = a_5 = \frac{1}{2}$ and G-partial metric space is replaced with metric space then (2.1) reduces to (1.3).

We also established the existence of a unique fixed point for a generalized weak C-contractive mapping in ordered G-partial metric spaces.

Theorem 2.3 : Let $(X, \preceq)$ be a partially ordered set and suppose that there exists a G-partial metric on X such that (X, G_p) is complete. Let $T : X \to X$ be continuous nondecreasing mapping. Suppose that for comparable $x, y \in X$, we have

$$\begin{aligned} \psi(G_p(Tx,Ty,Ty)) \quad \le \quad & \varphi(a_1G_p(x,y,y) + a_2G_p(x,Tx,Tx) + a_3G_p(y,Ty,Ty) \\ & +a_4G_p(x,Ty,Ty) + a_5G_p(y,Tx,Tx)) - \phi(G_p(x,y,y), \\ & G_p(x,Tx,Tx), G_p(y,Ty,Ty), G_p(x,Ty,Ty), \\ & G_p(y,Tx,Tx)) \end{aligned} \tag{2.2}$$

where a_1, a_2, a_3, a_4, $a_5 \in [0,1)$, $\sum_{i=1}^{5} a_i < 1$, and ψ, φ are altering distance functions with

$$\psi(t) - \varphi(t) \ge 0. \tag{2.3}$$

for $t \ge 0$, and $\phi : [0,\infty)^5 \to [0,\infty)$ is a continuous function with $\phi(v,w,x,y,z) = 0$ if and only if $v = w = x = y = z = 0$. If there exists $x_0 \in X$ such that $x_0 \preceq Tx_0$, then T has a fixed point.

Proof: Observe that if T satisfies (2.2) then it satisfies

$$\begin{aligned}\psi(G_p(Tx,Ty,Ty)) \quad \leq \quad & \varphi(aG_p(x,y,y)+bG_p(x,Tx,Tx)+bG_p(y,Ty,Ty) \\ & +cG_p(x,Ty,Ty)+cG_p(y,Tx,Tx))-\phi(G_p(x,y,y), \\ & G_p(x,Tx,Tx),G_p(y,Ty,Ty),G_p(x,Ty,Ty), \\ & G_p(y,Tx,Tx)) \end{aligned} \tag{2.4}$$

where $a = a_1$, $2b = a_2 + a_3$, $2c = a_4 + a_5$, $a + 2b + 2c < 1$ and $2b + 2c < 1$. We use (2.4) for our argument.

Let $x_0 \in X$ be arbitrarily chosen. Suppose $x_0 = Tx_0$ then x_0 is the fixed point of T. Let $x_0 \preceq Tx_0$, $x_1 \in X$ can be chosen such that $Tx_0 = x_1$. Since T is nondecreasing function, then $x_0 \preceq x_1 = Tx_0 \preceq x_2 = Tx_1 \preceq x_3 = Tx_2$.

Continuing the process, a sequence $\{x_n\}$ can be constructed such that $x_{n+1} = Tx_n$ with $x_0 \preceq x_1 \preceq x_2 \preceq x_3 \preceq ... \preceq x_n \preceq x_{n+1} ...$.

If $G_p(x_n, x_{n+1}, x_{n+1}) = 0$ for some $n \in N$ then T has a fixed point. Letting $G_p(x_n, x_{n+1}, x_{n+1}) > 0$ for all $n \in N$, we claim that

$$G_p(x_n,x_{n+1},x_{n+1}) \leq G_p(x_{n-1},x_n,x_n), n \in N \tag{2.5}$$

Suppose $x_n \neq x_{n+1}$, $G_p(x_n, x_{n+1}, x_{n+1}) > G_p(x_{n-1}, x_n, x_n)$ for some n_0 then

$$G_p(x_{n_0},x_{n_0+1},x_{n_0+1}) > G_p(x_{n_0-1},x_{n_0},x_{n_0}). \tag{2.6}$$

From (2.4) and (2.6) the proof of the claim is established as:

$$\begin{aligned}&\psi(G_p(x_{n_0},x_{n_0+1},x_{n_0+1})) = \psi(G_p(Tx_{n_0-1},Tx_{n_0},Tx_{n_0})) \\ &\leq \varphi(aG_p(x_{n_0-1},x_{n_0},x_{n_0})+bG_p(x_{n_0-1},Tx_{n_0-1},Tx_{n_0-1})+bG_p(x_{n_0},Tx_{n_0},Tx_{n_0}) \\ &+cG_p(x_{n_0-1},Tx_{n_0},Tx_{n_0})+cG_p(x_{n_0},Tx_{n_0-1},Tx_{n_0-1}))-\phi(G_p(x_{n_0-1},x_{n_0},x_{n_0}), \\ &G_p(x_{n_0-1},Tx_{n_0-1},Tx_{n_0-1}),G_p(x_{n_0},Tx_{n_0},Tx_{n_0}),G_p(x_{n_0-1},Tx_{n_0},Tx_{n_0}), \\ &G_p(x_{n_0},Tx_{n_0-1},Tx_{n_0-1}))\end{aligned}$$

$$\begin{aligned}&= \varphi(aG_p(x_{n_0-1},x_{n_0},x_{n_0})+bG_p(x_{n_0-1},x_{n_0},x_{n_0})+bG_p(x_{n_0},x_{n_0+1},x_{n_0+1}) \\ &+cG_p(x_{n_0-1},x_{n_0+1},x_{n_0+1})+cG_p(x_{n_0},x_{n_0},x_{n_0})) \\ &-\phi(G_p(x_{n_0-1},x_{n_0},x_{n_0}),G_p(x_{n_0-1},x_{n_0},x_{n_0}),G_p(x_{n_0},x_{n_0+1},x_{n_0+1}), \\ &\quad G_p(x_{n_0-1},x_{n_0+1},x_{n_0+1}),G_p(x_{n_0},x_{n_0},x_{n_0}))\end{aligned}$$

$$\begin{aligned}&\leq \varphi(aG_p(x_{n_0-1},x_{n_0},x_{n_0})+bG_p(x_{n_0-1},x_{n_0},x_{n_0})+bG_p(x_{n_0},x_{n_0+1},x_{n_0+1}) \\ &+cG_p(x_{n_0-1},x_{n_0},x_{n_0})+cG_p(x_{n_0},x_{n_0+1},x_{n_0+1})-cG_p(x_{n_0},x_{n_0},x_{n_0}) \\ &+cG_p(x_{n_0},x_{n_0},x_{n_0}))-\phi(G_p(x_{n_0-1},x_{n_0},x_{n_0}),G_p(x_{n_0-1},x_{n_0},x_{n_0}), \\ &\quad G_p(x_{n_0},x_{n_0+1},x_{n_0+1}),G_p(x_{n_0-1},x_{n_0+1},x_{n_0+1}),G_p(x_{n_0},x_{n_0},x_{n_0}))\end{aligned}$$

$$\begin{aligned}&\leq \varphi(aG_p(x_{n_0-1},x_{n_0},x_{n_0})+bG_p(x_{n_0-1},x_{n_0},x_{n_0})+bG_p(x_{n_0},x_{n_0+1},x_{n_0+1}) \\ &+cG_p(x_{n_0-1},x_{n_0},x_{n_0})+cG_p(x_{n_0},x_{n_0+1},x_{n_0+1}))-\phi(G_p(x_{n_0-1},x_{n_0},x_{n_0}), \\ &\quad G_p(x_{n_0-1},x_{n_0},x_{n_0}),G_p(x_{n_0},x_{n_0+1},x_{n_0+1}),G_p(x_{n_0-1},x_{n_0+1},x_{n_0+1}), \\ &\quad G_p(x_{n_0},x_{n_0},x_{n_0}))\end{aligned}$$

$$\begin{aligned}&\leq \varphi((a+2b+2c)max\{G_p(x_{n_0-1},x_{n_0},x_{n_0}),G_p(x_{n_0},x_{n_0+1},x_{n_0+1}))\} \\ &-\phi(G_p(x_{n_0-1},x_{n_0},x_{n_0}),G_p(x_{n_0-1},x_{n_0},x_{n_0}),G_p(x_{n_0},x_{n_0+1},x_{n_0+1}), \\ &\quad G_p(x_{n_0-1},x_{n_0+1},x_{n_0+1}),G_p(x_{n_0},x_{n_0},x_{n_0}))\end{aligned}$$

$$\leq \varphi(max\{G_p(x_{n_0-1}, x_{n_0}, x_{n_0}), G_p(x_{n_0}, x_{n_0+1}, x_{n_0+1}))\}$$
$$- \phi(G_p(x_{n_0-1}, x_{n_0}, x_{n_0}), G_p(x_{n_0-1}, x_{n_0}, x_{n_0}), G_p(x_{n_0}, x_{n_0+1}, x_{n_0+1}),$$
$$G_p(x_{n_0-1}, x_{n_0+1}, x_{n_0+1}), G_p(x_{n_0}, x_{n_0}, x_{n_0}))$$

$$\leq \varphi(G_p(x_{n_0}, x_{n_0+1}, x_{n_0+1})) - \phi(G_p(x_{n_0-1}, x_{n_0}, x_{n_0}), G_p(x_{n_0-1}, x_{n_0}, x_{n_0}),$$
$$G_p(x_{n_0}, x_{n_0+1}, x_{n_0+1}), G_p(x_{n_0-1}, x_{n_0+1}, x_{n_0+1}), G_p(x_{n_0}, x_{n_0}, x_{n_0})). \tag{2.7}$$

Using (2.4), (2.7) becomes

$$\phi(G_p(x_{n_0-1}, x_{n_0}, x_{n_0}), G_p(x_{n_0-1}, x_{n_0}, x_{n_0}), G_p(x_{n_0}, x_{n_0+1}, x_{n_0+1}),$$
$$G_p(x_{n_0-1}, x_{n_0+1}, x_{n_0+1}), G_p(x_{n_0}, x_{n_0}, x_{n_0})) = 0. \tag{2.8}$$

By property of ϕ, (2.8) yields

$$G_p(x_{n_0-1}, x_{n_0}, x_{n_0}) = 0,\ G_p(x_{n_0-1}, x_{n_0}, x_{n_0}) = 0,\ G_p(x_{n_0}, x_{n_0+1}, x_{n_0+1}) = 0,$$
$$G_p(x_{n_0-1}, x_{n_0+1}, x_{n_0+1}) = 0,\ G_p(x_{n_0}, x_{n_0}, x_{n_0}) = 0. \tag{2.9}$$

Since

$$\psi(G_p(x_{n_0}, x_{n_0+1}, x_{n_0+1})) = \psi(G_p(Tx_{n_0-1}, Tx_{n_0}, Tx_{n_0}))$$
$$\leq \varphi(aG_p(x_{n_0-1}, x_{n_0}, x_{n_0}) + bG_p(x_{n_0-1}, Tx_{n_0-1}, Tx_{n_0-1}) + bG_p(x_{n_0}, Tx_{n_0}, Tx_{n_0})$$
$$+ cG_p(x_{n_0-1}, Tx_{n_0}, Tx_{n_0}) + cG_p(x_{n_0}, Tx_{n_0-1}, Tx_{n_0-1})) - \phi(G_p(x_{n_0-1}, x_{n_0}, x_{n_0}),$$
$$G_p(x_{n_0-1}, Tx_{n_0-1}, Tx_{n_0-1}), G_p(x_{n_0}, Tx_{n_0}, Tx_{n_0}), G_p(x_{n_0-1}, Tx_{n_0}, Tx_{n_0}),$$
$$G_p(x_{n_0}, Tx_{n_0-1}, Tx_{n_0-1}))$$
$$= \varphi(aG_p(x_{n_0-1}, x_{n_0}, x_{n_0}) + bG_p(x_{n_0-1}, x_{n_0}, x_{n_0}) + bG_p(x_{n_0}, x_{n_0+1}, x_{n_0+1})$$
$$+ cG_p(x_{n_0-1}, x_{n_0+1}, x_{n_0+1}) + cG_p(x_{n_0}, x_{n_0}, x_{n_0}))$$
$$- \phi(G_p(x_{n_0-1}, x_{n_0}, x_{n_0}), G_p(x_{n_0-1}, x_{n_0}, x_{n_0}), G_p(x_{n_0}, x_{n_0+1}, x_{n_0+1}),$$
$$G_p(x_{n_0-1}, x_{n_0+1}, x_{n_0+1}), G_p(x_{n_0}, x_{n_0}, x_{n_0}))$$

$$\leq \varphi(aG_p(x_{n_0-1}, x_{n_0}, x_{n_0}) + bG_p(x_{n_0-1}, x_{n_0}, x_{n_0}) + bG_p(x_{n_0}, x_{n_0+1}, x_{n_0+1})$$
$$+ cG_p(x_{n_0-1}, x_{n_0}, x_{n_0}) + cG_p(x_{n_0}, x_{n_0+1}, x_{n_0+1}) - cG_p(x_{n_0}, x_{n_0}, x_{n_0})$$
$$+ cG_p(x_{n_0}, x_{n_0}, x_{n_0})) - \phi(G_p(x_{n_0-1}, x_{n_0}, x_{n_0}), G_p(x_{n_0-1}, x_{n_0}, x_{n_0}),$$
$$G_p(x_{n_0}, x_{n_0+1}, x_{n_0+1}), G_p(x_{n_0-1}, x_{n_0+1,x_{n_0+1}}), G_p(x_{n_0}, x_{n_0}, x_{n_0}))$$

$$\leq \varphi(aG_p(x_{n_0-1}, x_{n_0}, x_{n_0}) + bG_p(x_{n_0-1}, x_{n_0}, x_{n_0}) + bG_p(x_{n_0}, x_{n_0+1}, x_{n_0+1})$$
$$+ cG_p(x_{n_0-1}, x_{n_0}, x_{n_0}) + cG_p(x_{n_0}, x_{n_0+1}, x_{n_0+1}))$$
$$- \phi(G_p(x_{n_0-1}, x_{n_0}, x_{n_0}), G_p(x_{n_0-1}, x_{n_0}, x_{n_0}), G_p(x_{n_0}, x_{n_0+1}, x_{n_0+1}),$$
$$G_p(x_{n_0-1}, x_{n_0+1}, x_{n_0+1}), G_p(x_{n_0}, x_{n_0}, x_{n_0})). \tag{2.10}$$

Putting (2.9) into (2.10) yields

$$\psi(G_p(x_{n_0}, x_{n_0+1}, x_{n_0+1})) = 0. \tag{2.11}$$

By the property of ψ, (2.10) implies that $G_p(x_{n_0}, x_{n_0+1}, x_{n_0+1}) = 0$ which contradict $G_p(x_{n_0}, x_{n_0+1}, x_{n_0+1}) > 0$ for all $n \in N$, hence (2.5) holds. Thus, $\{G_p(x_n, x_{n+1}, x_{n+1})\}$ is a decreasing sequence, hence there exists $k \geq 0$ such that $lim_{n\to\infty} G_p(x_n, x_{n+1}, x_{n+1}) = k$.

Using (2.4), we obtain

$$\psi(G_p(x_{n+1}, x_{n+2}, x_{n+2})) = \psi(G_p(Tx_n, Tx_{n+1}, Tx_{n+1}))$$
$$\leq \varphi(a(G_p(x_n, x_{n+1}, x_{n+1}) + b(G_p(x_n, Tx_n, Tx_n) + b(G_p(x_{n+1}, Tx_{n+1}, Tx_{n+1})$$
$$+ c(G_p(x_n, Tx_{n+1}, Tx_{n+1}) + c(G_p(x_{n+1}, Tx_n, Tx_n)) - \phi(G_p(x_n, x_{n+1}, x_{n+1}),$$
$$G_p(x_n, Tx_n, Tx_n), G_p(x_{n+1}, Tx_{n+1}, Tx_{n+1}), G_p(x_n, Tx_{n+1}, Tx_{n+1}),$$
$$G_p(x_{n+1}, Tx_n, Tx_n))$$
$$= \varphi(a(G_p(x_n, x_{n+1}, x_{n+1}) + b(G_p(x_n, x_{n+1}, x_{n+1}) + b(G_p(x_{n+1}, x_{n+2}, x_{n+2})$$
$$+ c(G_p(x_n, x_{n+2}, x_{n+2}) + c(G_p(x_{n+1}, x_{n+1}, x_{n+1})) - \phi(G_p(x_n, x_{n+1}, x_{n+1}),$$

$$G_p(x_n, x_{n+1}, x_{n+1}), G_p(x_{n+1}, x_{n+2}, x_{n+2}), G_p(x_n, x_{n+2}, x_{n+2}), G_p(x_{n+1}, x_{n+1}, x_{n+1}))$$

$$\leq \varphi(a(G_p(x_n, x_{n+1}, x_{n+1}) + b(G_p(x_n, x_{n+1}, x_{n+1}) + b(G_p(x_{n+1}, x_{n+2}, x_{n+2}) + c(G_p(x_n, x_{n+1}, x_{n+1}) + c(G_p(x_{n+1}, x_{n+2}, x_{n+2}) - cG_p(x_{n+1}, x_{n+1}, x_{n+1}) + c(G_p(x_{n+1}, x_{n+1}, x_{n+1})) - \phi(G_p(x_n, x_{n+1}, x_{n+1}), G_p(x_n, x_{n+1}, x_{n+1}), G_p(x_{n+1}, x_{n+2}, x_{n+2}), G_p(x_n, x_{n+2}, x_{n+2}), G_p(x_{n+1}, x_{n+1}, x_{n+1}))$$

$$\leq \varphi(a(G_p(x_n, x_{n+1}, x_{n+1}) + b(G_p(x_n, x_{n+1}, x_{n+1}) + b(G_p(x_{n+1}, x_{n+2}, x_{n+2}) + c(G_p(x_n, x_{n+1}, x_{n+1}) + c(G_p(x_{n+1}, x_{n+2}, x_{n+2})) - \phi(G_p(x_n, x_{n+1}, x_{n+1}), G_p(x_n, x_{n+1}, x_{n+1}), G_p(x_{n+1}, x_{n+2}, x_{n+2}), G_p(x_n, x_{n+2}, x_{n+2}), G_p(x_{n+1}, x_{n+1}, x_{n+1}))$$

$$\leq \varphi((a + 2b + 2c)max\{G_p(x_n, x_{n+1}, x_{n+1}), G_p(x_{n+1}, x_{n+2}, x_{n+2})\}) - \phi(G_p(x_n, x_{n+1}, x_{n+1}), G_p(x_n, x_{n+1}, x_{n+1}), G_p(x_{n+1}, x_{n+2}, x_{n+2}), G_p(x_n, x_{n+2}, x_{n+2}), G_p(x_{n+1}, x_{n+1}, x_{n+1}))$$

$$\leq \varphi(max\{G_p(x_n, x_{n+1}, x_{n+1}), G_p(x_{n+1}, x_{n+2}, x_{n+2})\}) - \phi(G_p(x_n, x_{n+1}, x_{n+1}), G_p(x_n, x_{n+1}, x_{n+1}), G_p(x_{n+1}, x_{n+2}, x_{n+2}), G_p(x_n, x_{n+2}, x_{n+2}), G_p(x_{n+1}, x_{n+1}, x_{n+1}))$$

$$\leq \varphi(G_p(x_{n+1}, x_{n+2}, x_{n+2})) - \phi(G_p(x_n, x_{n+1}, x_{n+1}), G_p(x_n, x_{n+1}, x_{n+1}), G_p(x_{n+1}, x_{n+2}, x_{n+2}), G_p(x_n, x_{n+2}, x_{n+2}), G_p(x_{n+1}, x_{n+1}, x_{n+1})). \quad (2.12)$$

Using (2.3) we have,

$$\phi(G_p(x_n, x_{n+1}, x_{n+1}), G_p(x_n, x_{n+1}, x_{n+1}), G_p(x_{n+1}, x_{n+2}, x_{n+2}), G_p(x_n, x_{n+2}, x_{n+2}), G_p(x_{n+1}, x_{n+1}, x_{n+1})) = 0.$$

Taking the limit as $n \to \infty$ in the above inequality yields

$$lim\ inf_{n\to\infty}\ (\phi(G_p(x_n, x_{n+1}, x_{n+1}), G_p(x_n, x_{n+1}, x_{n+1}), G_p(x_{n+1}, x_{n+2}, x_{n+2}), G_p(x_n, x_{n+2}, x_{n+2}), G_p(x_{n+1}, x_{n+1}, x_{n+1}))) = 0.$$

By the continunity of ϕ we have

$$\phi(lim\ inf_{n\to\infty}G_p(x_n, x_{n+1}, x_{n+1}), lim\ inf_{n\to\infty}G_p(x_n, x_{n+1}, x_{n+1}), lim\ inf_{n\to\infty}G_p(x_{n+1}, x_{n+2}, x_{n+2}), lim\ inf_{n\to\infty}G_p(x_n, x_{n+2}, x_{n+2}), lim\ inf_{n\to\infty}G_p(x_{n+1}, x_{n+1}, x_{n+1})) = 0.$$

The property of ϕ gives that

$$lim\ inf_{n\to\infty}G_p(x_n, x_{n+1}, x_{n+1}) = 0,\ lim\ inf_{n\to\infty}G_p(x_{n+1}, x_{n+2}, x_{n+2}) = 0,$$
$$lim\ inf_{n\to\infty}G_p(x_n, x_{n+2}, x_{n+2}) = 0,\ lim\ inf_{n\to\infty}G_p(x_{n+1}, x_{n+1}, x_{n+1})) = 0. \quad (2.13)$$

Taking the inferior limit in (2.12) and using (2.13), $\psi(k) = 0$, this implies that $k = 0$. Therefore $lim_{n\to\infty}G_p(x_n, x_{n+1}, x_{n+1}) = 0$.

Now we claim that $\{x_n\}$ is a Cauchy sequence. It is sufficient to show that $\{x_{2n}\}$ is a Cauchy sequence. On the contrary, suppose $\{x_{2n}\}$ is not a Cauchy sequence then there exists $\epsilon > 0$ and two subsequences $\{x_{2n_k}\}$ and $\{x_{2m_k}\}$ of $\{x_{2n}\}$ such that $n(k) > m(k) > k$ and sequences in (2.4) tend to ϵ as $k \to \infty$. For two comparable elements $y = x_{2n_k+1}$ and $x = x_{2m_k}$ we can get, from (2.3) that

$$\begin{aligned}
&\psi(G_p(x_{2n_k+1}, x_{2m_k}, x_{2m_k})) = \psi(G_p(Tx_{2n_k}, Tx_{2m_k-1}, Tx_{2m_k-1})) \\
&\quad \leq \varphi(aG_p(x_{2n_k}, x_{2m_k-1}, x_{2m_k-1}) + bG_p(x_{2n_k}, Tx_{2n_k}, Tx_{2n_k}) \\
&\quad + bG_p(x_{2m_k-1}, Tx_{2m_k-1}, Tx_{2m_k-1}) + cG_p(x_{2n_k}, Tx_{2m_k-1}, Tx_{2m_k-1}) \\
&\quad + cG_p(x_{2m_k-1}, Tx_{2n_k}, Tx_{2n_k})) - \phi(G_p(x_{2n_k}, x_{2m_k-1}, x_{2m_k-1}), \\
&\quad\; G_p(x_{2n_k}, Tx_{2n_k}, Tx_{2n_k}), G_p(x_{2m_k-1}, Tx_{2m_k-1}, Tx_{2m_k-1}), \\
&\quad\; G_p(x_{2n_k}, Tx_{2m_k-1}, Tx_{2m_k-1}), G_p(x_{2m_k-1}, Tx_{2n}, Tx_{2n_k})) \\
&\quad \leq \varphi(aG_p(x_{2n_k}, x_{2m_k-1}, x_{2m_k-1}) + bG_p(x_{2n_k}, x_{2n_k+1}, x_{2n_k+1}) \\
&\quad + bG_p(x_{2m_k-1}, x_{2m_k}, x_{2m_k}) + cG_p(x_{2n_k}, x_{2m_k}, x_{2m_k}) + cG_p(x_{2m_k-1}, x_{2n+1}, x_{2n_k+1})) \\
&\quad - \phi(G_p(x_{2n_k}, x_{2m_k-1}, x_{2m_k-1}), G_p(x_{2n_k}, x_{2n_k+1}, x_{2n_k+1}), G_p(x_{2m_k-1}, x_{2m_k}, x_{2m_k}), \\
&\quad G_p(x_{2n_k}, x_{2m_k}, x_{2m_k}), G_p(x_{2m_k-1}, x_{2n+1}, x_{2n_k+1})). \qquad (2.14)
\end{aligned}$$

As $k \to \infty$ in (2.14), we get

$$\psi(\epsilon) \leq \varphi(\epsilon) - \phi(\epsilon, \epsilon, \epsilon, \epsilon, \epsilon),$$

This implies that $\phi(\epsilon, \epsilon, \epsilon, \epsilon, \epsilon) = 0$, hence $\epsilon = 0$, a contradiction. Thus $\{x_{2n}\}$ is a Cauchy sequence and so is $\{x_n\}$. Since (X, G_p) is complete then the sequence $\{x_n\}$ converges to some $z \in X$, that is $lim_{n\to\infty} G_p(x_n, z, z) = 0$ and $lim_{n\to\infty} G_p(x_n, z, z) = lim_{n\to\infty} G_p(x_n, x_n, x_n) = lim_{n\to\infty} G_p(x_n, x_m, x_m)$
$= lim_{n\to\infty} G_p(z, z, z) = 0$.

Applying the rectangle inequality, we have

$$G_p(z, Tz, Tz) \leq G_p(z, x_n, x_n) + G_p(x_n, , Tz, Tz) - G_p(x_n, x_n, x_n) \leq G_p(z, x_n, x_n) + G_p(Tx_{n-1}, Tz, Tz).$$

Taking $n \to \infty$ in the above inequalities, with the continunity of T and Lemma 1.9 give that $G_p(z, Tz, Tz) \leq G_p(Tz, Tz, Tz)$.
By $G_P(1)$, $G_p(z, Tz, Tz) \geq G_p(Tz, Tz, Tz)$. This implies that

$$G_p(z, Tz, Tz) = G_p(Tz, Tz, Tz) \qquad (2.15)$$

By combining (2.4) and (2.15), we have

$$\begin{aligned}
\psi(G_p(z, Tz, Tz)) &= \psi(G_p(Tz, Tz, Tz)) \\
&\leq \varphi(aG_p(z, z, z) + bG_p(z, Tz, Tz) + bG_p(z, Tz, Tz) + cG_p(z, Tz, Tz) \\
&+ cG_p(z, Tz, Tz)) \\
&\quad - \phi(G_p(z, z, z), G_p(z, Tz, Tz), G_p(z, Tz, Tz), G_p(z, Tz, Tz), G_p(z, Tz, Tz)) \\
&\quad = \varphi((2b + 2c)G_p(z, Tz, Tz)) - \phi(G_p(z, z, z), G_p(z, Tz, Tz), G_p(z, Tz, Tz), \\
&\quad G_p(z, Tz, Tz), G_p(z, Tz, Tz)) \\
&\leq \varphi(G_p(z, Tz, Tz)) - \phi(G_p(z, z, z), G_p(z, Tz, Tz), G_p(z, Tz, Tz), \\
&\quad G_p(z, Tz, Tz), G_p(z, Tz, Tz)),
\end{aligned}$$

$$\begin{aligned}
&\phi(G_p(z, z, z), G_p(z, Tz, Tz), G_p(z, Tz, Tz), G_p(z, Tz, Tz), G_p(z, Tz, Tz)) \\
&\qquad \leq \varphi(G_p(z, Tz, Tz)) - \psi(G_p(z, Tz, Tz)) = 0.
\end{aligned}$$

Thus $G_p(z, Tz, Tz) = 0$, hence $z = Tz$. Therefore z is a fixed point of T.

Remarks 2.4: Theorem 2.1 is more general than Theorem 2.1 of Chen and Zhu [6] because G-partial metric space generalized partial metric space and the weakly C-contractive map of Chen and Zhu is included in our map. Also the result generalizes the results of Choudhury [9] in terms of space and maps. Generalized weakly C-contractive maps is more general than the usual Hardy and Rogers contractive map and our space also generalized the usual metric space, therefore Theorem 2.1 generalizes Theorem 2.1 of Olaleru [13].

Corollary 2.5: Let $(X, \preceq)$ be a partially ordered set and suppose that there exists a G-partial metric on X such that (X, G_p) is complete. Let $T : X \to X$ be continuous nondecreasing mapping. Suppose that for comparable $x, y \in X$, we have

$$\begin{aligned} \psi(G_p(Tx, Ty, Ty)) \quad &\leq \quad \varphi(\frac{G_p(x, Ty, Ty) + G_p(y, Tx, Tx)}{2}) \\ &\quad -\phi(G_p(x, Ty, Ty), G_p(y, Tx, Tx)) \end{aligned} \tag{2.16}$$

$$\text{where } \psi(t) - \varphi(t) \geq 0 \tag{2.17}$$

for all $t \geq 0$, and $\phi : [0, \infty)^2 \to [0, \infty)$ is a continuous function with $\phi(y, z) = 0$ if and only if $y = z = 0$. If there exists $x_0 \in X$ such that $x_0 \preceq Tx_0$ then T has a fixed point.

Corollary 2.6 : Let $(X, \preceq)$ be a partially ordered set and suppose that there exists a G-partial metric on X such that (X, G_p) is complete. Let $T : X \to X$ be continuous nondecreasing mapping. Suppose that for comparable $x, y \in X$, we have

$$\psi(G_p(Tx, Ty, Ty)) \quad \leq \quad \varphi(G_p(x, y, y)) - \phi(G_p(x, y, y)) \tag{2.18}$$

$$\psi(t) - \varphi(t) \geq 0 \tag{2.19}$$

for all $t \geq 0$, and $\phi : [0, \infty) \to [0, \infty)$ is a continuous function with $\phi(x) = 0$ if and only if $x = 0$. If there exists $x_0 \in X$ such that $x_0 \preceq Tx_0$ then T has a fixed point.

The proof of the corollary follows from Theorem 2.1.

Remarks 2.7: Corollary 2.5 is an analog result of Chen and Zhu [6] from partial metric space to ordered G-partial metric space. If we replace ordered G-partial metric space with G-metric space and $\psi(k) = k$, $\varphi(t) = t$ in (2.14) then corollary 2.6 gives Theorem 2.1 of Aage and Saluke [14].

Example 2.8: Let$X = [0, 14]$ be endowed with a G-partial metric $G_p : X \times X \times X \to R^+$ defined by $G_p(x, y, y) = max\{x, y, y\}$. Clearly, we can show that the G-partial metric space (X, G_p) is complete. Also, we define the mapping $T : X \to X$ by $Tx = \frac{x}{3}$. Let us take $\psi, \varphi : [0, +\infty) \to [0, +\infty)$ such that $\psi(t) = \frac{t^2}{9}$ and $\varphi(t) = \frac{t^2}{3}$, respectively, and take $\phi : [0, +\infty)^5 \to [0, +\infty)$ such that $\phi(u, v, x, y, z) = \frac{(u+v+x+y+z)^2}{9}$. If $x \geq y$ then

$G_p(Tx, Ty, Ty) = max\{\frac{x}{3}, \frac{y}{3}, \frac{y}{3}\} = \frac{x}{3}$.

By simple calculation we have,

$$G_p(Tx, Ty, Ty) \leq \tfrac{1}{3} G_p(x, y, y), \tag{2.20}$$

$$G_p(Tx, Ty, Ty) \leq \tfrac{1}{3}[G_p(x, Tx, Tx) + G_p(y, Ty, Ty)], \tag{2.21}$$

$$G_p(Tx, Ty, Ty) \leq \tfrac{1}{3}[G_p(x, Ty, Ty) + G_p(y, Tx, Tx)]. \tag{2.22}$$

Also,

$$\begin{aligned} &G_p(x, y, y) + G_p(x, Tx, Tx) + G_p(y, Ty, Ty) + G_p(x, Ty, Ty) + G_p(y, Tx, Tx) \\ &\quad = G_p(x, y, y) + G_p(x, \tfrac{x}{3}, \tfrac{x}{3}) + G_p(y, \tfrac{y}{3}, \tfrac{y}{3}) + G_p(x, \tfrac{y}{3}, \tfrac{y}{3}) + G_p(y, \tfrac{x}{3}, \tfrac{x}{3}) \\ &\quad = max\{x, y, y\} + max\{x, \tfrac{x}{3}, \tfrac{x}{3}\} + G_p(y, \tfrac{y}{3}, \tfrac{y}{3}) + max\{x, \tfrac{y}{3}, \tfrac{y}{3}\} + G_p(y, \tfrac{x}{3}, \tfrac{x}{3}) \\ &\quad = 3x + G_p(y, \tfrac{y}{3}, \tfrac{y}{3}) + G_p(y, \tfrac{x}{3}, \tfrac{x}{3}). \end{aligned}$$

Hence,

$\psi(G_p(Tx, Ty, Ty)) = \frac{x^2}{9} \leq \frac{(3x \ + G_p(y,\ \frac{y}{3},\ \frac{y}{3}) + \ G_p(y,\ \frac{x}{3},\ \frac{x}{3}))^2}{9}$

$$
\begin{aligned}
&\leq \tfrac{(3x\ +G_p(y,\ \frac{y}{3},\ \frac{y}{3})+G_p(y,\ \frac{x}{3},\ \frac{x}{3}))^2}{3} \\
&- \tfrac{(3x\ +G_p(y,\ \frac{y}{3},\ \frac{y}{3})+G_p(y,\ \frac{x}{3},\ \frac{x}{3}))^2}{9} \\
&= \varphi(a_1G_p(x,y,y) + a_2G_p(x,Tx,Tx) + a_3G_p(y,Ty,Ty) + a_4G_p(x,Ty,Ty) \\
&+ a_5G_p(y,Tx,Tx)) - \phi(G_p(x,y,y), G_p(x,Tx,Tx), G_p(y,Ty,Ty), \\
&\quad G_p(x,Ty,Ty), G_p(y,Tx,Tx)).
\end{aligned}
$$

If $y \geq x$ then we have,
$G_p(Tx,Ty,Ty) = max\{\frac{x}{3}, \frac{y}{3}, \frac{y}{3}\} = \frac{y}{3}$.

Also,

$$
\begin{aligned}
&G_p(x,y,y) + G_p(x,Tx,Tx) + G_p(y,Ty,Ty) + G_p(x,Ty,Ty) + G_p(y,Tx,Tx) \\
&\quad = G_p(x,y,y) + G_p(x,\tfrac{x}{3},\tfrac{x}{3}) + G_p(y,\tfrac{y}{3},\tfrac{y}{3}) + G_p(x,\tfrac{y}{3},\tfrac{y}{3}) + G_p(y,\tfrac{x}{3},\tfrac{x}{3}) \\
&\quad = max\{x,y,y\} + G_p(x,\tfrac{x}{3},\tfrac{x}{3}) + max\{y,\tfrac{y}{3},\tfrac{y}{3}\} + G_p(x,\tfrac{y}{3},\tfrac{y}{3}) + max\{y,\tfrac{x}{3},\tfrac{x}{3}\} \\
&\quad = 3y + G_p(x,\tfrac{x}{3},\tfrac{x}{3}) + G_p(x,\tfrac{y}{3},\tfrac{y}{3}).
\end{aligned}
$$

Therefore,

$$
\psi(G_p(Tx,Ty,Ty)) = \tfrac{y^2}{9} \leq \tfrac{(3y\ +G_p(x,\ \frac{x}{3},\ \frac{x}{3})+G_p(x,\ \frac{y}{3},\ \frac{y}{3}))^2}{9}
$$

$$
\begin{aligned}
&\leq \tfrac{(3y\ +G_p(x,\ \frac{x}{3},\ \frac{x}{3})+G_p(x,\ \frac{y}{3},\frac{y}{3}))^2}{3} - \tfrac{(3y\ +G_p(x,\ \frac{x}{3},\frac{x}{3})+G_p(x,\ \frac{y}{3},\frac{y}{3})^2}{9} \\
&= \varphi(a_1G_p(x,y,y) + a_2G_p(x,Tx,Tx) + a_3G_p(y,Ty,Ty) + a_4G_p(x,Ty,Ty) \\
&+ a_5G_p(y,Tx,Tx)) - \phi(G_p(x,y,y), G_p(x,Tx,Tx), G_p(y,Ty,Ty), \\
&\quad G_p(x,Ty,Ty), G_p(y,Tx,Tx)).
\end{aligned}
$$

From the above argument, we conclude that (2.2) holds. Therefore all the conditions of Theorem 2.1 is satisfied. The fixed point of T is 0.

3 Conclusions

The class of generalized weakly C-contractive mappings is introduced to G-partial metric spaces. Some fixed point results for these maps are proved in ordered G-partial metric spaces. Examples is given to support our result. The introduction of these contractive maps will open up new research area for interested researchers in fixed point theory and applications.

Competing Interests

The author declares that no competing interests exist.

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